The Home Forum®

A brief look at infinity

WHAT IS INFINITY?

If the first thing that came to your mind was the amount of chocolate you'd like to eat or the number of weeks you'd like to have of summer vacation, then you are on the right track.

But if you thought of a very big number – a million, a trillion, a quadrillion – then we need to have a little talk.

Infinity is not the same thing as "bigness." It is above and beyond "big." A big number is no more infinite than a small number. Five million is no more infinite than five. Why? Because both are finite (FYE-nite). They are a certain amount, and no more.

As mathematicians define it, infinity is more like a summer vacation that never ends. In an infinite vacation, you would go to bed each night knowing that you'd have at least one more day off.

Infinity is a thing (let's avoid the word "number" for the moment) that is bigger than any number. It is bigger than 5, bigger than 500, bigger than 5 million. We represent that thing by the mathematical symbol ∞. (The man who came up with that symbol is described below.)

There is an infinity of counting numbers (the counting numbers are 1,2,3, and so on). If you try to reach the limit of the counting numbers, you can't. There's always one more, and one more after that. Even if you count up to one google (a google is the number 1 followed by 100 zeros), you will find that there is a number beyond it (a google and one) and a second one close on its heels (a google and two), and on and on.

ARISTOTLE WAS SUSPICIOUS OF THE WHOLE IDEA – AND IT *IS* STRANGE, WHEN YOU EXAMINE IT.

We can use the infinity symbol in arithmetic, but the arithmetic of infinity is weird. For example:

$$\infty + 1 = \infty$$

Why is this? Because we know that infinity is bigger than any number. Adding 1 to infinity cannot change that property of infinity.

It is also true that:

$$\infty - 1 = \infty$$

Again, when you take 1 away from infinity, the result is still larger than any limit. That's what infinity means: Something that's not finite, not limited. As applied to numbers, it means "very big" as well as "very small" – and everything else, too.

The truly weird part occurs when you subtract infinity from infinity. You can get any result you please! (Just the kind of question you'd like to have on math test, don't you think?) You can decide the correct answer because all answers are correct. You can make:

$$\infty - \infty = 0$$

You can also make it equal to 1 or 7 or 19.

Here's how: Let's return to the counting numbers (1,2,3, etc.). If you get rid of all of them, you will have nothing. Therefore:

$$\infty - \infty = 0$$

However, you might decide to subtract all the counting numbers starting with 8, leaving seven numbers (1, 2, 3, 4, 5, 6, and 7). You have removed an infinity of counting numbers, so you the result is:

$$\infty - \infty = 7$$

But the strangest result of all is:

$$\infty - \infty = \infty$$

How could that be? Again, start with the counting numbers. There are an infinite number of even numbers: 2, 4, 6, 8, and so on. (If you don't believe me, just try to think of an upper limit for the even numbers. You can't; there isn't one.) If you start with the counting numbers, which are infinite, and remove just the even numbers, which are also infinite, you are left with the odd numbers, which are – that's right – infinite. Therefore, you can correctly state that:

$$\infty - \infty = \infty$$

Think of it this way: Infinite does not mean "every number that exists." There are, in fact, an infinite number of infinite numbers. Infinity can be small, too. Start with the numeral 1, for example Divide it in half. Divide it in half again (1/4). And again (1/8). Keep going. What will you find? That there are an infinite number of fractions between 1 and 0.

For another strange example of "transfinite arithmetic" (that is, the arithmetic of infinity), read about the circle-and-line diagram on the next page.

The concept of infinity undergirds a lot of modern mathematics, even practical mathematics. For example: Suppose you are stretching a clothesline across your backyard. You want to know how tightly you need to stretch it to prevent it from sagging under the weight of wet laundry. The easiest way to analyze this problem mathematically is to pretend that the clothesline is made up of an infinite number of pieces, each of them infinitesimal (that is, infinitely short) in length. This analysis produces a very simple (well, for a mathematician) equation to describe the curve of the line.

We should end our discussion of infinity with English poet William Blake. Blake has perhaps the most famous statement about infinity in his poem "Auguries of Innocence," written about 1801:

To see a world in a grain of sand
And a heaven in a wild flower,
Hold infinity in the palm of your hand
And eternity in an hour.

Our hands are relatively small things. They can hold only a few pieces of chocolate or grasp a small number of coins. But they can mark today as vacation. And tomorrow. And the day after that. And so we can continue reaching toward infinity.

David Alan Grier

A sideways symbol

THE concept of infinity was known to the ancient Greeks, though Aristotle found the idea somewhat suspect. He called it "imperfect, unfinished, unlimited, and therefore unthinkable."

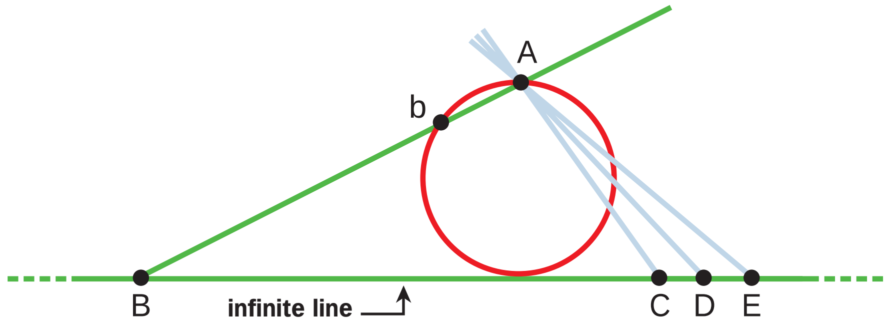
The symbol for infinity, an elongated figure 8 lying on its side (∞), was invented by English mathematician John Wallis (1616-1703), shown here. Wallis first used the symbol in a book titled "Arithmetica Infinitorum" ("The Arithmetic of Infinitesimals") in 1655. He was probably the best-known English mathematician before Sir Isaac Newton (1642-1727) of falling-apple fame.

Wallis was also a cryptographer, a maker and breaker of codes. He decoded messages for pro-Parliamentarian rebels during the English Civil Wars (1642-51).

D.A.G.

KAREN SCHNEIDER – STAFF

An infinite line meets a finite circle



Mathematicians use this diagram to prove that a circle is just an infinite line that's been scrunched up into a ball.

Which is bigger?

LOOK at the two geometric figures pictured above, a circle and an infinite line. Which one is bigger?

Most people would say that the infinite line is bigger. It's longer than the circle. But mathematicians see it another way. They can prove that, for every point on the infinite line, there is a corresponding point on the circle. And for every point on the circle, there is a corresponding point on the line. In mathematical terms, then, a circle is just an infinite line that's been scrunched up into a ball.

Remember, we're talking about an ideal circle and line – figures not drawn with pencil, ruler, and compass. The points on these figures are as small as they need to be. Infinitely small, in fact.

Look at the diagram. The circle touches the line at only one point, as if it were the sun sitting on the horizon at sunset. Point A is directly opposite the point where the circle touches the line. From any point you choose on the line (B, C, D, E), draw a line through point A.

Any line you draw crosses the circle at exactly one point before it hits Point A. If you're a mathematician, you're well on your way to proving that for each point on the infinite line there is a corresponding point on the circle, and vice versa. (We can't explain the proof here – it has to do with the unique angles described by the lines drawn through Point A.) But that fact is enough to prove that the circle is as infinite as the line.

D.A.G.

Speaking of big numbers...

CAN you fold a sheet of newspaper in half 10 times? Newsprint is thin, and a sheet of it is rather large, so that should be easy, right?

Go ahead, try it with this newspaper. (Make sure everyone's read it first.)

How many folds did you make?

Let me guess – eight? Maybe nine? That's the most we could manage, too.

What if you start with a bigger sheet of newspaper? You can try it, but you'll get the same result – nine folds, no more. To understand why, you need to know something about exponential growth.

When you fold the paper once, the number of layers of newspaper doubles to two. Fold it again, and it doubles again, to four (2 x 2). Three folds makes it eight layers thick (2 x 2 x 2); four folds, 16 layers (2 x 2 x 2 x 2 – see where this is going?). Five folds is 32. Six, 64. Seven: 128; eight: 256; nine: 512; and (whew!) ten: 1,024!

No wonder! On the tenth fold, no matter how big the sheet of paper is to begin with, you are trying to fold 512 layers to make 1,024 layers. Each time you fold, the number of layers multiplies by 2. Another way to say this is that the layers increase by a *factor* of 2.

As your paper-folding shows, doubling an amount will get you to large numbers quickly! Scientists use this concept of exponential growth when studying things like populations.

The global human population, for example, has experienced exponential growth over time. Since the first modern humans appeared, it took 2.5 million years to reach a population of 1 billion, around 1800. It took a mere 130 years to double the population to 2 billion in 1930. And 44 more to reach 4 billion in 1974.

'EXPONENTIAL GROWTH' PRODUCES BIG NUMBERS FAST. PROVE IT WITH THIS NEWSPAPER.

Our doubling rate has slowed a bit since then. The world's population is about 6.2 billion today and isn't likely to reach the 8 billion mark (twice the 1974 population) until around 2030.

Sara Steindorf

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TODAY'S ARTICLE ON CHRISTIAN SCIENCE

Bringing a spiritual perspective to daily life

The new/old normal

IT WAS A TURNING POINT. I knew my thinking had to change, but I was confused. How I had faced challenges in the past did not measure up to the new demand. I was really tormented. Then I read this verse in the Bible: "When I was a child, I spake as a child, I understood as a child, I thought as a child: but when I became a man, I put away childish things" (I Cor. 13:11).

I still remember my feeling of relief. My shoulders relaxed, and I knew I wouldn't betray earlier convictions by changing my modus operandi. It was all right to grow. I didn't have to hold on to old ways. Changing the way I looked at things and how I acted did not necessarily indicate that I had been wrong. It just meant that I had to embrace a higher standard for thinking and acting. In short, circumstances were calling for more mature, less self-centered reasoning.

This important change from old ways of thinking and acting has come to my memory recently as we hear about the "new normal." This new normal was replacing the old norms of materialism, indifference to family, and taking safety for granted. There is a recognition that a new normal exists but also that the "old normal" is creeping back.

Loving as deeply and universally as we can today, we are raising the standard for generations to come.

There seems to be a consensus that the new normal prevails when it comes to being alert, even a bit edgy, about vulnerability to terrorist attacks. What is slipping back into the old norm has to do with something deeper. Immediately after the tragedy, many people took a good look at what was really important to them and adopted a higher standard for their lives. As time passes, though, this new normal appears to be losing significance.

The question now is, How can we keep from falling back into the inferior old ways. The answer, I feel, is to see what underlies the rush of a higher idealism. I believe that most of us know deep in our hearts that the good lived in loving our families, loving our neighbors, loving and helping those less fortunate than ourselves, is the norm we want to achieve.

Mary Baker Eddy, in founding this newspaper, set a high standard, or norm, when she wrote, "The object of the Monitor is to injure no man, but to bless all mankind" ("The First Church of Christ, Scientist, and Miscellany," pg. 353). This object was the purpose of her life, and one might safely say the real goal of most of our lives, too. Living as a good family member, a good neighbor, a good citizen of the world, will continue to lift our standards and be that which we consider to be normal.

It is axiomatic that there is really no status quo in human behavior. As we confront the world and evil, we are either going up higher or falling back. There is a great need for an invariable, spiritual normative on which a progressive standard can be placed.

Mrs. Eddy, who earlier had discovered Christian Science, wrote, "According to Christian Science, perfection is normal, – not miraculous" ("Miscellaneous Writings 1883–1896," pg. 104). Reasoning from the assumption of there being one good, all powerful God, we naturally assume that His/Her creation is perfect.

Such assumption becomes absolute conviction as we get clearer and fuller views of God. We gain this clear view through heartfelt prayer and a willingness to change, to be Godlike.

Our lives are uplifted by living in accord with this understanding of Deity, the divine and supreme ruler. Naturally our attitudes change. We put away childish things. We mature. The verse quoted at the beginning of this article is preceded by, "But when that which is perfect is come, then that which is in part shall be done away" (I Cor. 13:10).

Old norms based only partly on God's love will yield to ever higher standards as we gain a fuller understanding and acceptance of the God that is Love. Loving as deeply and universally as we can today, we are raising the standard of behavior for generations to come. This has to raise the norm of human behavior so that peace in the world will be natural and eternal.

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Now we see through a glass, darkly; but then face to face: now I know in part; but then shall I know even as also I am known.

I Corinthians 13:12